Worcester County Mathematics League

Varsity Meet 4 - March 4, 2020

COACHES' COPY
ROUNDS, ANSWERS, AND SOLUTIONS

Worcester County Mathematics League
Varsity Meet 4 - March 4, 2020
Answer Key

Round 1 - Elementary Number Theory

1. $2^5 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11$
2. $3140_{(5)}$
3. $9432, 9036, 9936$ (need all three)

Round 2 - Algebra I

1. $32a^3b^6$
2. $k = 6$
3. \$1.50 or 150 cents (must use proper notation)

Round 3 - Geometry

1. 37
2. 5
3. $\dfrac{\pi r^2}{12} - \dfrac{\sqrt{3}r^2}{12}$ or $\dfrac{r^2}{12}(\pi - \sqrt{3})$

Round 4 - Logs, Exponents, and Radicals

1. $x = -7$
2. $x = 4$ (only)
3. 9

Round 5 - Trigonometry

1. $\dfrac{\sqrt{2} - \pi}{4}$
2. $2\sqrt{39}$
3. $1, -1$, and $\dfrac{1}{4}$ (need all three)

Team Round

1. $\dfrac{16\pi\sqrt{2}}{3}$
2. 31 cents or \$0.31
3. no solution
4. 240
5. $\dfrac{1}{x}$
6. 120°
7. 16
8. $|\sin(3)|, |\sin(4)|, |\sin(1)|, |\sin(2)|$ (must have absolute value symbols)
9. $\dfrac{11\pi}{24}$

Worcester County Mathematics League
Varsity Meet 4 - March 4, 2020
Round 1 - Elementary Number Theory

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Give the prime factorization of $554,400$.

2. Determine the least common multiple of $10100_{(2)}$ and $101010_{(2)}$ and express it in base five.

3. Find all four-digit numbers which can be written as $9A3B$ (where A and B represent the hundreds and units digits, respectively) and are divisible by 36.

ANSWERS

(1 pt) 1. ______________________

(2 pts) 2. ______________________

(3 pts) 3. ______________________

Shepherd Hill, Shepherd Hill, St. John's

Worcester County Mathematics League
Varsity Meet 4 - March 4, 2020
Round 2 - Algebra I

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Simplify the following expression:

$$(2ab^2)^3 + (2ab^2)^2(6ab^2)$$

2. If the vertices of parallelogram $ABCD$ are $A(-2,3), B(1,2), C(3,5)$, and $D(0,k)$, find k.

3. Ella buys a certain number of tomato plants for \$9. Brian buys two more plants than Ella, but of a different variety (pepper plants) for \$10. If ten tomato plants and four pepper plants cost \$20, what does one tomato plant cost?

ANSWERS

(1 pt) 1. ______________________

(2 pts) 2. ______________________

(3 pts) 3. ______________________

Tantasqua, Shrewsbury, Bromfield

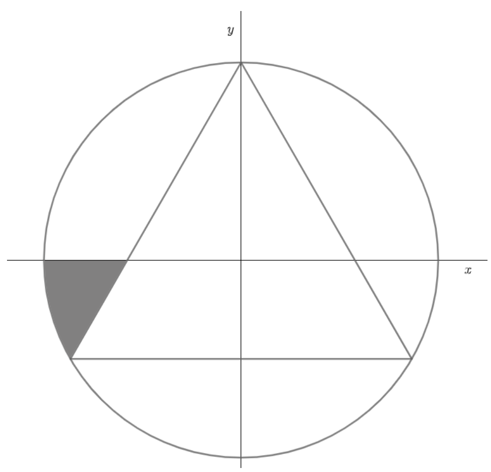
Worcester County Mathematics League
Varsity Meet 4 - March 4, 2020
Round 3 - Geometry

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. The lengths of the sides of a non-isosceles triangle are 7, x, and 18. If x is an integer, what is the smallest possible perimeter of the triangle?

2. Parallelogram $ABCD$ has a perimeter of 32 with $AB = 4x + z$, $BC = 6x + 3z$, $CD = 2y + 1$ and $AD = 3y$. Find $x + y$.

3. An equilateral triangle is inscribed in a circle of radius r such that an altitude of the triangle lies along the y-axis. Find the area (in terms of r) bounded by the triangle, the circle, and the x-axis in quadrant 3.



ANSWERS

(1 pt) 1. ____________________

(2 pts) 2. ____________________

(3 pts) 3. ____________________

Wachusett, Bromfield, Shrewsbury

Worcester County Mathematics League
Varsity Meet 4 - March 4, 2020
Round 4 - Logs, Exponents, and Radicals

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Solve for x.

$$\sqrt{\left(\frac{1}{4}\right)^3} \cdot \sqrt[3]{8^{-4}} = 2^x$$

2. Solve for x.

$$2\log_4 x + \log_2(x-3) = 2$$

3. Compute n^5 if

$$\log_{4n} 96 = \log_{5n} 75\sqrt{5}.$$

ANSWERS

(1 pt) 1. ____________________

(2 pts) 2. ____________________

(3 pts) 3. ____________________

Sutton, Groton-Dunstable, Algonquin

Worcester County Mathematics League
Varsity Meet 4 - March 4, 2020
Round 5 - Trigonometry

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Evaluate the following expression. All angles should be expressed in radians. Leave your answer as a single fraction.

$$\sin\left(\frac{5\pi}{4}\right)\cdot\cos\left(\frac{2\pi}{3}\right)+\arctan(-1)$$

2. Consider quadrilateral $ABCD$ where $\measuredangle DAB = \measuredangle ABC = 120°$. If $AD = 8, AB = 6$, and $BC = 4$, determine the length of $\overline{CD}$.

3. Compute all possible values of $\cos x$ if

$$3^{\tan x} = 81^{\sin x}.$$

ANSWERS

(1 pt) 1. ________________

(2 pts) 2. ________________

(3 pts) 3. ________________

QSC, Bromfield, Algonquin

Worcester County Mathematics League

Varsity Meet 4 - March 4, 2020

Team Round

All answers must be in simplest exact form in the answer section.

NO CALCULATORS ALLOWED

1. A $120°$ sector is cut from a circle of radius 6 and bent to form the lateral surface area of a cone. What is the volume of the cone?

2. In Numismia, there are only two types of currency: 5 cent coins and 9 cent coins. What is the largest amount of money that cannot be made with a combination of those two coins?

3. Solve for x.
$$\sqrt{4-11x} - x + 2 = 0$$

4. Semicircles are drawn on each side of a triangle. Their areas are 25π, 144π, and 169π. Find the area of the triangle.

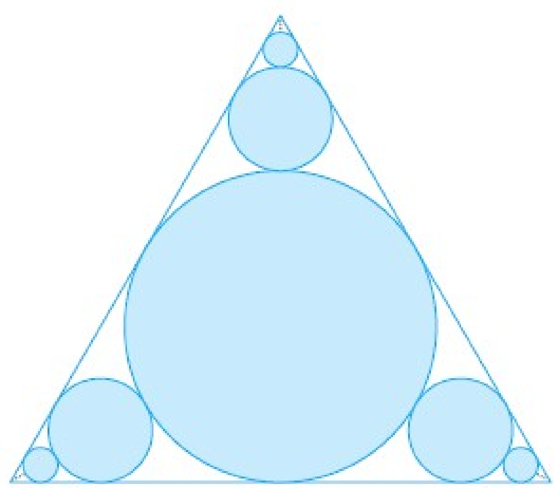
5. If
$$a^{2x} - 2a^x - 3 = 0$$
where $a > 1$ and $x > 0$, find the value of $\log_3 a$ in terms of x.

6. Determine the size of the largest angle in a triangle having sides of $1.5, 2.5$, and 3.5.

7. The sum of a real number, its square, and its square root is 276. Find the number.

8. Order the following from least to greatest. Note: *angles are in radians.*
$$|\sin(1)|, |\sin(2)|, |\sin(3)|, |\sin(4)|$$

9. A circle is inscribed in an equilateral triangle with sides of length 2. Three more triangles are placed inside the triangle such that each is tangent to two sides of the triangle and a previously inscribed circle. This process continues forever. Determine the collective area of the circles.



Worcester County Mathematics League
Varsity Meet 4 - March 4, 2020
Team Round Answer Sheet

ANSWERS

1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________

5. ______________________________

6. ______________________________

7. ______________________________

8. ______________________________

9. ______________________________

Sutton, Tantasqua, Tahanto, Groton-Dunstable, Westborough, Doherty, Bromfield, QSC, St. John's

Worcester County Mathematics League
Varsity Meet 4 - March 4, 2020
Answer Key

Round 1 - Elementary Number Theory

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2. $3140_{(5)}$
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2. $k = 6$
3. \$1.50 or 150 cents (must use proper notation)

Round 3 - Geometry

1. 37
2. 5
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Round 4 - Logs, Exponents, and Radicals

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4. 240
5. $\dfrac{1}{x}$
6. 120°
7. 16
8. $|\sin(3)|, |\sin(4)|, |\sin(1)|, |\sin(2)|$ (must have absolute value symbols)
9. $\dfrac{11\pi}{24}$

Round 1 - Elementary Number Theory

1. Give the prime factorization of $554,400$.

Solution: First, break the number down into $5544 \cdot 100 = 5544 \cdot 2^2 \cdot 5^2$. Since the digits of 5544 sum to 9, it's divisible by 9: $616 \cdot 3^2 \cdot 2^2 \cdot 5^2$. Since the last two digits of 616 are divisible by 4, the whole number is divisible by 4: $154 \cdot 3^2 \cdot 2^4 \cdot 5^2$. Dividing 154 by 2 gives us 77, which breaks down into 7 times 11. The full factorization: $\boxed{2^5 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11}$.

2. Determine the least common multiple of $10100_{(2)}$ and $101010_{(2)}$ and express it in base five.

Solution: First, we put both numbers into base ten. Since $10100_{(2)}$ represents 1 2^4 and 1 2^2, the first number is 20. The second number represents 1 of each of 2^5, 2^3, and 2^1, which sums to 42. Since

$$20 = 2^2 \cdot 5$$

and

$$42 = 2 \cdot 3 \cdot 7,$$

to match factors we need to arrive at $2^2 \cdot 3 \cdot 5 \cdot 7 = 420$.

Now we must put this number into base 5. We look at non-negative integer powers of 5 (1, 5, 25, 125) and see that 125 (or 5^3) divides into 420 three times with remainder 55. We see that 25 (or 5^2) divides into 55 once with remainder 20. Then 5 (or 5^1) divides evenly into 20 four times with no remainder. Therefore, $420_{(10)} = \boxed{3140_{(5)}}$.

3. Find all four-digit numbers which can be written as $9A3B$ (where A and B represent the hundreds and units digits, respectively) and are divisible by 36.

Solution: For $9A3B$ to be divisible by 36, it must be divisible by 9 and by 4.

To be divisible by 9, the sum of the digits must itself be divisible by 9, so $9 + A + 3 + B$ will either sum to 18 or 27. This leads us to find $A + B$ to be equal to 6 or 15.

To be divisible by 4, the last two digits must be divisible by 4. That means B must be 2 or 6.

We make a small chart:

If...	$A + B = 6$	$A + B = 15$
$B = 2$	$A = 4$	not possible
$B = 6$	$A = 0$	$A = 9$

Our possible combinations lead us to the numbers $\boxed{9432, 9036, 9936}$.

Round 2 - Algebra I

1. Simplify the following expression:

$$(2ab^2)^3 + (2ab^2)^2(6ab^2)$$

Solution:

$$(2ab^2)^3 + (2ab^2)^2(6ab^2)$$
$$(2^3a^3b^6) + (4a^2b^4)(6ab^2)$$
$$(8a^3b^6) + (24a^3b^6)$$
$$\boxed{32a^3b^6}$$

2. If the vertices of parallelogram $ABCD$ are $A(-2,3), B(1,2), C(3,5)$, and $D(0,k)$, find k.

Solution: For parallelogram $ABCD$, we know that $\overline{AB}$ must be parallel with $\overline{CD}$ and their slopes must be equal. Since

$$\text{slope of } \overline{AB} = \frac{3-2}{-2-1} = -\frac{1}{3}$$

we can find k by setting $-\frac{1}{3}$ equal to the slope of $\overline{CD}$:

$$\text{slope of } \overline{CD} = \frac{k-5}{0-3} = \frac{k-5}{-3} = -\frac{1}{3}$$
$$\frac{k-5}{-3} = -\frac{1}{3}$$
$$k-5=1$$
$$\boxed{k=6}$$

3. Ella buys a certain number of tomato plants for \$9. Brian buys two more plants than Ella, but of a different variety (pepper plants) for \$10. If ten tomato plants and four pepper plants cost \$20, what does one tomato plant cost?

Solution: Let n be the number of tomato plants bought at t dollars each. Thus,

$$nt = 9.$$

Let p be the cost of one pepper plant. Thus,

$$(n+2)p = 10.$$

We know that

$$10t + 4p = 20$$

and substitute from there:

$$10t + 4p = 20$$
$$10\left(\frac{9}{n}\right) + 4\left(\frac{10}{n+2}\right) = 20$$
$$\frac{9}{n} + \frac{4}{n+2} = 2$$
$$9(n+2) + 4(n) = 2(n)(n+2)$$

$$9n + 18 + 4n = 2n^2 + 4n$$

$$0 = 2n^2 - 9n - 18$$

$$0 = (2n + 3)(n - 6)$$

The only valid value for n is 6, so $(6)p = 9$ and one pepper plant costs $\frac{9}{6} = \boxed{\$1.50}$.

Round 3 - Geometry

1. The lengths of the sides of a non-isosceles triangle are 7, x, and 18. If x is an integer, what is the smallest possible perimeter of the triangle?

Solution: For any triangle, the sum of the lengths of the two smaller sides must be larger than the length of the longest side. To make the perimeter as small as possible, we want x to be as small as possible, and therefore we treat 18 as the largest side.

Since $7 + x > 18$ and x is an integer length, the smallest integer to cause the inequality to be true is 12. Therefore, the smallest perimeter is $7 + 12 + 18 = \boxed{37}$

2. Parallelogram $ABCD$ has a perimeter of 32 with $AB = 4x + z$, $BC = 6x + 3z$, $CD = 2y + 1$ and $AD = 3y$. Find $x + y$.

Solution: In a parallelogram, opposite sides are equal:

$$4x + z = 2y + 1 \Longrightarrow 4x - 2y + z = 1$$

$$6x + 3z = 3y \Longrightarrow 6x - 3y + 3z = 0 \Longrightarrow= 2x - y + z = 0$$

Comparing the first equation to $\frac{2}{3}$ the second equation, we see

$$I : 4x - 2y + z = 1$$

$$\frac{2}{3}II : 4x - 2y + 2z = 0$$

Subtracting these equations gives us $z = -1$. Plugging in -1 for z, our equations are now identical: $4x - 2y = 2 \Longrightarrow 2x - y = 1$.

Knowing that all four sides sum to 32 gives us another equation. Summing and replacing z with -1:

$$4x - 1 + 6x - 3 + 2y + 1 + 3y = 32$$

$$10x + 9y = 35$$

To solve the system of

$$10x + 5y = 35$$
$$2x - y = 1$$

we multiply the second equation by 5 and subtract from the first:

$$10x + 5y = 35$$

$$-(10x - 5y = 5)$$

$$\overline{\qquad\qquad\qquad}$$

$$10y = 30$$

and $y = 3$. Plugging back in to the $2x - y = 1$ equation yields $x = 2$, so $x + y = \boxed{5}$.

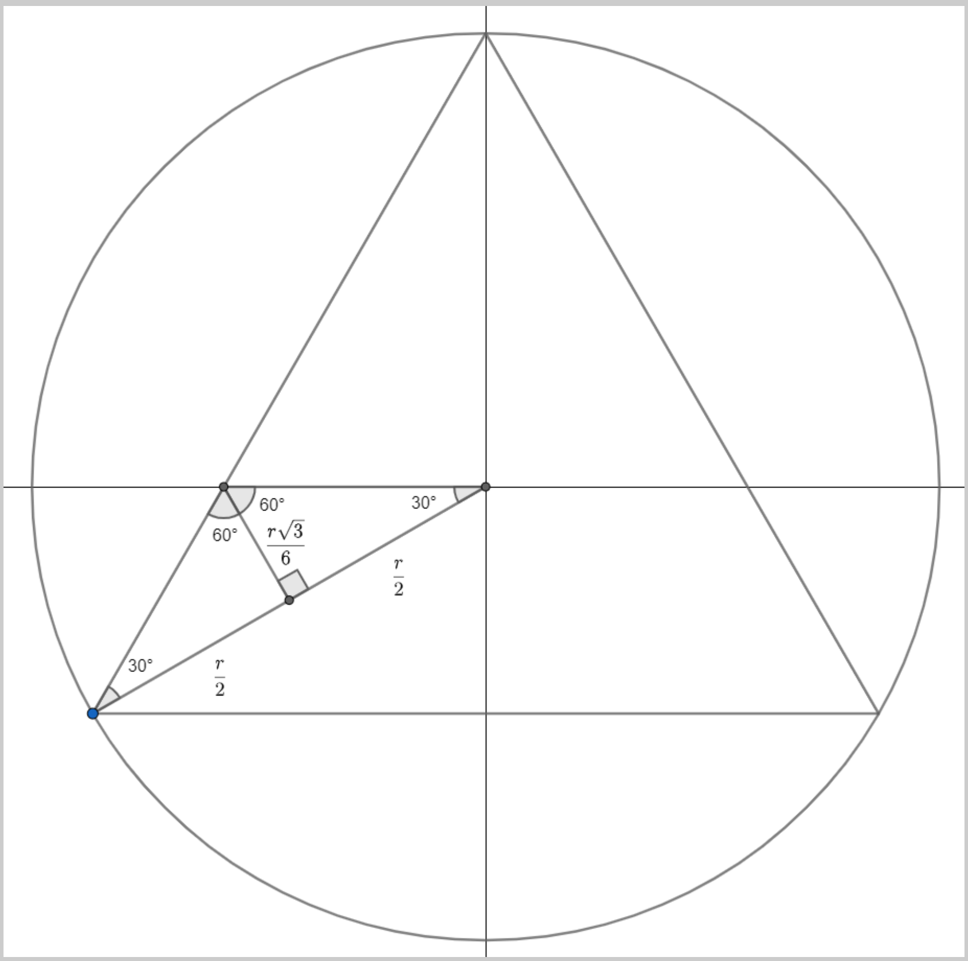
3. An equilateral triangle is inscribed in a circle of radius r such that an altitude of the triangle lies along the y-axis. Find the area (in terms of r) bounded by the triangle, the circle, and the x-axis in quadrant 3.

Solution: The area we seek is the difference between the area of the sector and the area of the triangle.

To find the area of the sector, we note that a radius drawn to the corner of the triangle results in creating two $30°$ angles, and therefore the measure of the sector is $30°$ via alternate interior angles. Since $30°$ is one-twelfth of the area of the entire circle, the sector has area $\frac{\pi r^2}{12}$.

To find the area of the triangle, we note that it is isosceles. Dropping an altitude cuts the radius in half and creates two $30°-60°-90°$ triangles. This leads us to find the altitude is $\frac{r\sqrt{3}}{6}$ and the area of the triangle to be $\frac{1}{2}(r)\left(\frac{r\sqrt{3}}{6}\right) = \frac{r^2\sqrt{3}}{12}$.

The difference is $\frac{\pi r^2}{12} - \frac{r^2\sqrt{3}}{12} = \boxed{\frac{r^2}{12}(\pi - \sqrt{3})}$.



Round 4 - Logs, Exponents, and Radicals

1. Solve for x.

$$\sqrt{\left(\frac{1}{4}\right)^3} \cdot \sqrt[3]{8^{-4}} = 2^x$$

Solution:

$$\sqrt{\left(\frac{1}{4}\right)^3} \cdot \sqrt[3]{8^{-4}} = 2^x$$

$$\sqrt{\frac{1}{64}} \cdot 2^{-4} = 2^x$$

$$\frac{1}{8} \cdot \frac{1}{16} = 2^x$$

$$\frac{1}{2^3 \cdot 2^4} = 2^x$$

$$2^{-7} = 2^x$$

So $\boxed{x = -7}$.

2. Solve for x.

$$2\log_4 x + \log_2(x-3) = 2$$

Solution: Using the change-of-base formula:

$$2\log_4 x + \log_2(x-3) = 2$$

$$2 \cdot \frac{\log_2 x}{\log_2 4} + \log_2(x-3) = 2$$

$$2 \cdot \frac{\log_2 x}{2} + \log_2(x-3) = 2$$

$$\log_2 x + \log_2(x-3) = 2$$

$$\log_2(x(x-3)) = 2$$

$$2^2 = x^2 - 3x$$

$$0 = x^2 - 3x - 4$$

$$0 = (x-4)(x+1)$$

So $x = 4$ or $x = -1$, but since letting $x = -1$ has undefined logarithms, our answer is $\boxed{x = 4}$.

3. Compute n^5 if

$$\log_{4n} 96 = \log_{5n} 75\sqrt{5}.$$

Solution: Let

$$\log_{4n} 96 = \log_{5n} 75\sqrt{5} = A$$

which gives us two separate equations:

$$\log_{4n} 96 = A \qquad \text{and} \qquad \log_{5n} 75\sqrt{5} = A.$$

We rearrange to find

$$(4n)^A = 96 \qquad \text{and} \qquad (5n)^A = 75\sqrt{5}.$$

Divide the second equation by the first to get

$$\frac{(5n)^A}{(4n)^A} = \frac{75\sqrt{5}}{96}$$

$$\left(\frac{5}{4}\right)^A = \frac{25\sqrt{5}}{32} = \frac{5^{\frac{5}{2}}}{4^{\frac{5}{2}}}$$

so $A = \frac{5}{2}$. Plugging back in to one of the equations,

$$(5n)^{\frac{5}{2}} = 75\sqrt{5}$$

$$(5n)^{\frac{5}{2}} = 3 \cdot 5^{\frac{5}{2}}$$

$$(5n)^5 = 3^2 \cdot 5^5$$

$$n^5 = 3^2 = \boxed{9}$$

Round 5 - Trigonometry

1. Evaluate the following expression. Leave your answer (in radians) as a single fraction.

$$\sin\left(\frac{5\pi}{4}\right)\cdot\cos\left(\frac{2\pi}{3}\right)+\arctan(-1)$$

Solution:

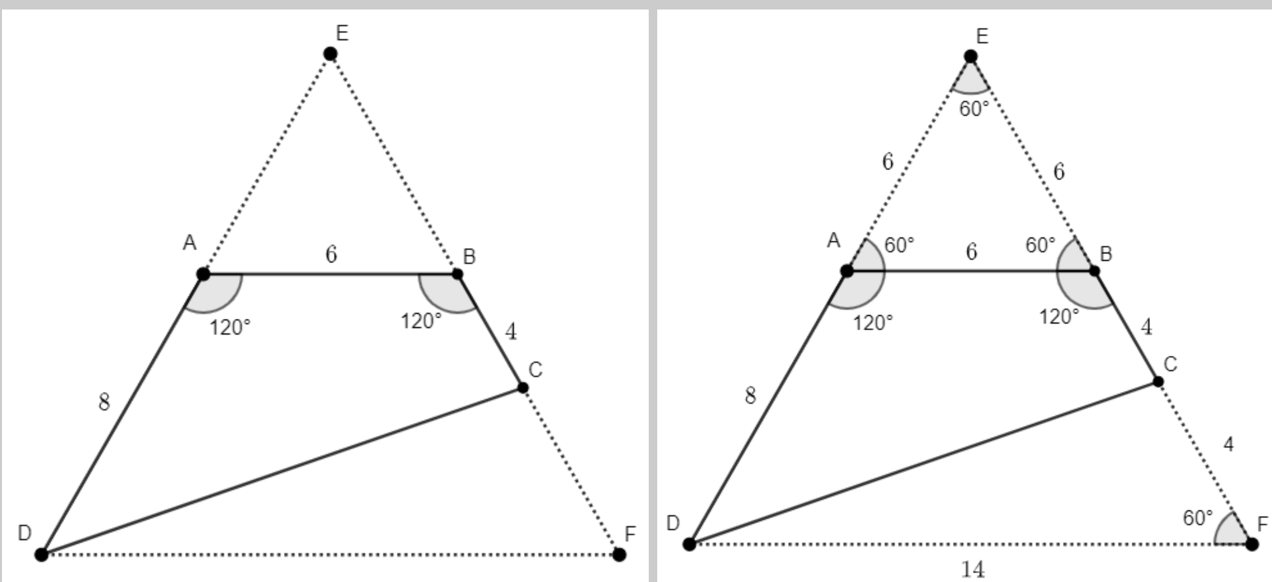
$$\left(-\frac{\sqrt{2}}{2}\right)\cdot\left(-\frac{1}{2}\right)+\left(-\frac{\pi}{4}\right)$$

$$\frac{\sqrt{2}}{4}-\frac{\pi}{4}$$

$$\boxed{\frac{\sqrt{2}-\pi}{4}}$$

2. Consider quadrilateral $ABCD$ where $\measuredangle DAB = \measuredangle ABC = 120^\circ$. If $AD = 8, AB = 6$, and $BC = 4$, determine the length of $\overline{CD}$.

Solution: Extend $\overline{DA}$ and $\overline{BC}$ to point E to form an equilateral triangle. Extend $\overline{BC}$ the other direction to F and connect F with D to create a 60° angle and a larger equilateral triangle (ΔDEF)



At this point, we can use the law of cosines with triangle ΔDEC or ΔDFC to determine the length of $\overline{CD}$. Here we will use ΔDFC.

$$DC^2 = 14^2 + 4^2 - 2\cdot 14\cdot 4\cdot \cos 60^\circ$$

$$DC^2 = 196 + 16 - 112\left(\frac{1}{2}\right)$$

$$DC^2 = 212 - 56)$$

$$DC^2 = 156$$

$$DC = \sqrt{156} = \boxed{2\sqrt{39}}$$

3. Compute all possible values of $\cos x$ if

$$3^{\tan x} = 81^{\sin x}.$$

Solution: Modifying the right-hand side we get

$$3^{\tan x} = 3^{4\sin x}$$

$$\tan x = 4\sin x$$

$$\frac{\sin x}{\cos x} - 4\sin x = 0$$

$$\sin x\left(\frac{1}{\cos x} - 4\right) = 0$$

We find that the equation is true if $\sin x = 0$ or $\cos x = \frac{1}{4}$. Since $\cos x = 1$ or -1 when $\sin x = 0$ the equation is true when $\boxed{\cos x = \pm 1 \text{ or } \frac{1}{4}}$.

Team Round

1. A 120° sector is cut from a circle of radius 6 which is then bent to form the lateral surface area of a cone. What is the volume of the cone?

Solution: A circle with radius 6 has circumference 12π. Since the 120° sector is one-third of the full circle, the cut sector has an arc length of 4π. This arc forms the circumference of the base of the newly formed cone.

If the cone's circumference is 4π, it's radius is 2. With the lateral height being 6 (the former circle's radius), we find the height to be $\sqrt{32}$. The volume of this cone is therefore

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi 2^2\sqrt{32} = \frac{4\pi}{3} \cdot 4\sqrt{2} = \boxed{\frac{16\pi\sqrt{2}}{3}}.$$

2. In Numismia, there are only two types of currency: 5 cent coins and 9 cent coins. What is the largest amount of money that cannot be made with a combination of those two coins?

Solution: Let's begin by printing out a table of cents. We know that any multiple of 5 or 9 can be made, so we cross out all multiples of 5 and 9 to indicate this.

1	2	3	4	$\cancel{5}$	6	7	8	$\cancel{9}$	$\cancel{10}$
11	12	13	14	$\cancel{15}$	16	17	$\cancel{18}$	19	$\cancel{20}$
21	22	23	24	$\cancel{25}$	26	$\cancel{27}$	28	29	$\cancel{30}$
31	32	33	34	$\cancel{35}$	$\cancel{36}$	37	38	39	$\cancel{40}$
41	42	43	44	$\cancel{45}$	46	47	48	49	$\cancel{50}$
51	52	53	$\cancel{54}$	$\cancel{55}$	56	57	58	59	$\cancel{60}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

We also know that any time we add 9 cents to an already-made amount of money, that new amount of money can also be made. So from 5 cents, we can cancel out 14, 23, 32, 41. From 10, we can cancel out 19, 28, 37, and 46.

1	2	3	4	$\cancel{5}$	6	7	8	$\cancel{9}$	$\cancel{10}$
11	12	13	$\cancel{14}$	$\cancel{15}$	16	17	$\cancel{18}$	$\cancel{19}$	$\cancel{20}$
21	22	$\cancel{23}$	24	$\cancel{25}$	26	$\cancel{27}$	$\cancel{28}$	29	$\cancel{30}$
31	$\cancel{32}$	33	34	$\cancel{35}$	$\cancel{36}$	$\cancel{37}$	38	39	$\cancel{40}$
$\cancel{41}$	42	43	44	$\cancel{45}$	$\cancel{46}$	47	48	49	$\cancel{50}$
51	52	53	$\cancel{54}$	$\cancel{55}$	56	57	58	59	$\cancel{60}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Continuing this process, adding 9 cents to every already-made value gives us the following table. The largest number is below.

1	2	3	4	$\cancel{5}$	6	7	8	$\cancel{9}$	$\cancel{10}$
11	12	13	$\cancel{14}$	$\cancel{15}$	16	17	$\cancel{18}$	$\cancel{19}$	$\cancel{20}$
21	22	$\cancel{23}$	$\cancel{24}$	$\cancel{25}$	26	$\cancel{27}$	$\cancel{28}$	$\cancel{29}$	$\cancel{30}$
$\boxed{31}$	$\cancel{32}$	$\cancel{33}$	$\cancel{34}$	$\cancel{35}$	$\cancel{36}$	$\cancel{37}$	$\cancel{38}$	$\cancel{39}$	$\cancel{40}$
$\cancel{41}$	$\cancel{42}$	$\cancel{43}$	$\cancel{44}$	$\cancel{45}$	$\cancel{46}$	$\cancel{47}$	$\cancel{48}$	$\cancel{49}$	$\cancel{50}$
$\cancel{51}$	$\cancel{52}$	$\cancel{53}$	$\cancel{54}$	$\cancel{55}$	$\cancel{56}$	$\cancel{57}$	$\cancel{58}$	$\cancel{59}$	$\cancel{60}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

3. Solve for x.

$$\sqrt{4-11x}-x+2=0$$

Solution:

$$\sqrt{4-11x}-x+2=0$$
$$\sqrt{4-11x}=x-2$$
$$4-11x=x^2-4x+4$$
$$0=x^2+7x$$
$$0=x(x-7)$$

The numbers 0 and 7 are candidates to be solutions, but plugging in both of them yields issues:

$$\sqrt{4-11(0)}-0+2=4\neq 0$$
$$\sqrt{4-11(7)}-7+2=DNE$$

There are $\boxed{\text{no solutions}}$.

4. Semicircles are drawn on each side of a triangle. Their areas are 25π, 144π, and 169π. Find the area of the triangle.

Solution: The areas of each semicircle are given, so we double the areas, divide by π, and square root the value to arrive at the radius. The side with 25π has radius $\sqrt{50}=5\sqrt{2}$, the side with 144π has radius $\sqrt{288}=12\sqrt{2}$, and the side with 169π has radius $\sqrt{338}=13\sqrt{2}$.

Since the sides of the triangle would be diameters of each semicircle, the sides of the triangle are lengths $10\sqrt{2}, 24\sqrt{2}$, and $26\sqrt{2}$. These sides are in a ratio of 5:12:13, which forms a right triangle. We can use the legs to determine the area:

$$A=\frac{1}{2}(10\sqrt{2})(24\sqrt{2})=120\cdot 2=\boxed{240}.$$

5. If

$$a^{2x}-2a^x-3=0$$

where $a>1$ and $x>0$, find the value of $\log_3 a$ in terms of x.

Solution: We begin by solving for a^x:

$$(a^x)^2-2(a^x)-3=0$$
$$(a^x-3)(a^x+1)=0$$

Since a^x, a positive number raised to a positive power can never equal -1, the only option is $a^x=3$. From here, we note that $a=3^{1/x}$ and rewriting this in logarithm form is $\log_3 a=\boxed{\frac{1}{x}}$.

6. Determine the size of the largest angle in a triangle having sides of $1.5, 2.5$, and 3.5.

Solution: Make this easier to start, since a triangle with sides twice as long will have the same angles: $3, 5, 7$. We know the side across from the largest angle is 7, so we set up the law of cosines to calculate the angle:

$$7^2 = 3^2 + 5^2 - 2(3)(5)\cos\theta$$

$$\frac{49 - 9 - 25}{-30} = \cos\theta$$

$$\frac{15}{-30} = \cos\theta$$

$$-\frac{1}{2} = \cos\theta$$

The angle in the second quadrant that generates $\cos\theta = -\frac{1}{2}$ is $\boxed{\theta = \frac{2\pi}{3} = 120^\circ}$.

7. The sum of a real number, its square, and its square root is 276. Find the number.

Solution: Setting this up as an equation, we write

$$x + x^2 + \sqrt{x} = 276$$

and consider what types of numbers would be possible. We see that x must be a positive number (due to the square root) and we hope that x is a perfect square which would ensure that the sum of the terms was guaranteed to be an integer. We try 1, 4, 9, 16, 25, ...

$$1 + 1^2 + \sqrt{1} = 3$$

$$4 + 4^2 + \sqrt{4} = 22$$

$$9 + 9^2 + \sqrt{9} = 93$$

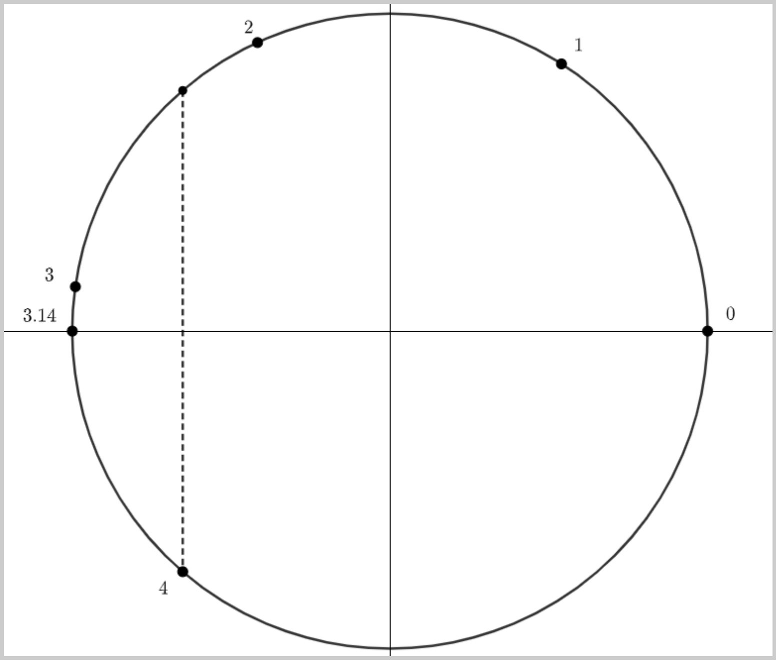
$$16 + 16^2 + \sqrt{16} = 276(!)$$

So $\boxed{x = 16}$.

8. Order the following from least to greatest. Note: *angles are in radians.*

$$|\sin(1)|, |\sin(2)|, |\sin(3)|, |\sin(4)|$$

Solution: First, plot the general locations of 1, 2, 3, and 4 radians around the unit circle.

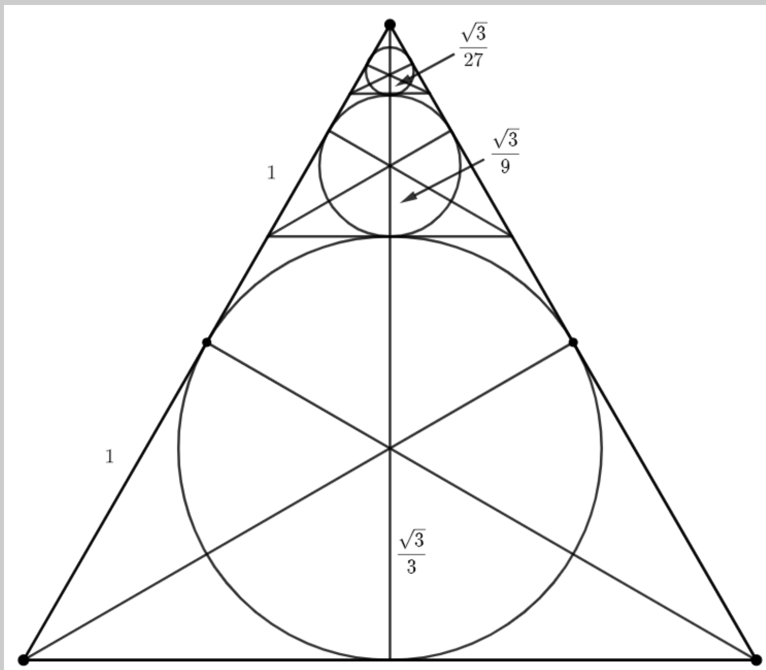


We see that $|\sin\theta|$ is the magnitude of the y-coordinate at θ. The farther away θ is from the x-axis, the larger $|\sin\theta|$ will be.

It's clear from the circle above that 3 radians is the closest to the x-axis (only .14 radians away). The next closest is 4 radians, which is $4 - 3.14 = .86$ radians away. 1 radian is 1 unit away (duh) and 2 radians is $.14 + 1 = 1.14$ radians away. Therefore, from smallest to largest: $\boxed{|\sin(3)|, |\sin(4)|, |\sin(1)|, |\sin(2)|}$.

9. A circle is inscribed in an equilateral triangle with sides of length 2. Three more triangles are placed inside the triangle such that each is tangent to two sides of the triangle and a previously inscribed circle. This process continues forever. Determine the collective area of the circles.

Solution: The equilateral triangle below, with side length 2, has height $\sqrt{3}$. The centroid breaks the medians into a $1:2$ ratio, so the big triangle has radius $\frac{\sqrt{3}}{3}$. Placing a single smaller circle into the space above the first will yield a situation that is a third of the original size, so the ratio of that circle will be $\frac{\sqrt{3}}{9}$. Continuing this, we see radii of $\frac{\sqrt{3}}{27}$, $\frac{\sqrt{3}}{81}$, and $\frac{\sqrt{3}}{243}$.



The total area can be represented by

$$A = \pi r_1^2 + 3\pi r_2^2 + 3\pi r_3^2 + 3\pi r_4^2 + 3\pi r_5^2 + \cdots$$

$$A = \pi r_1^2 + 3\pi r_2^2 + 3\pi\left(\frac{r_2}{3}\right)^2 + 3\pi\left(\frac{r_2}{9}\right)^2 + 3\pi\left(\frac{r_2}{27}\right)^2 + \cdots$$

$$A = \pi r_1^2 + 3\pi r_2^2\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \cdots\right)$$

$$A = \pi r_1^2 + 3\pi r_2^2\left(\frac{1}{1-\frac{1}{9}}\right)$$

$$A = \pi r_1^2 + 3\pi r_2^2 \cdot \frac{9}{8}$$

$$A = \pi\left(\frac{\sqrt{3}}{3}\right)^2 + 3\pi\left(\frac{\sqrt{3}}{9}\right)^2 \cdot \frac{9}{8}$$

$$A = \frac{\pi}{3} + \frac{\pi}{8}$$

$$A = \boxed{\frac{11\pi}{24}}$$